Detecting Vortices in a Structured Finite Element Model of High-Temperature Superconductors

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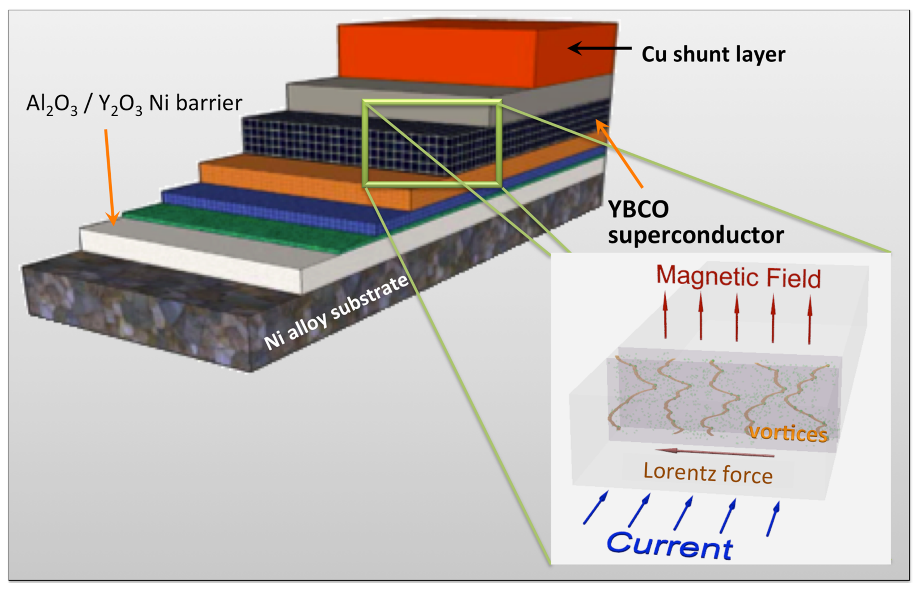
High Temperature Superconductors

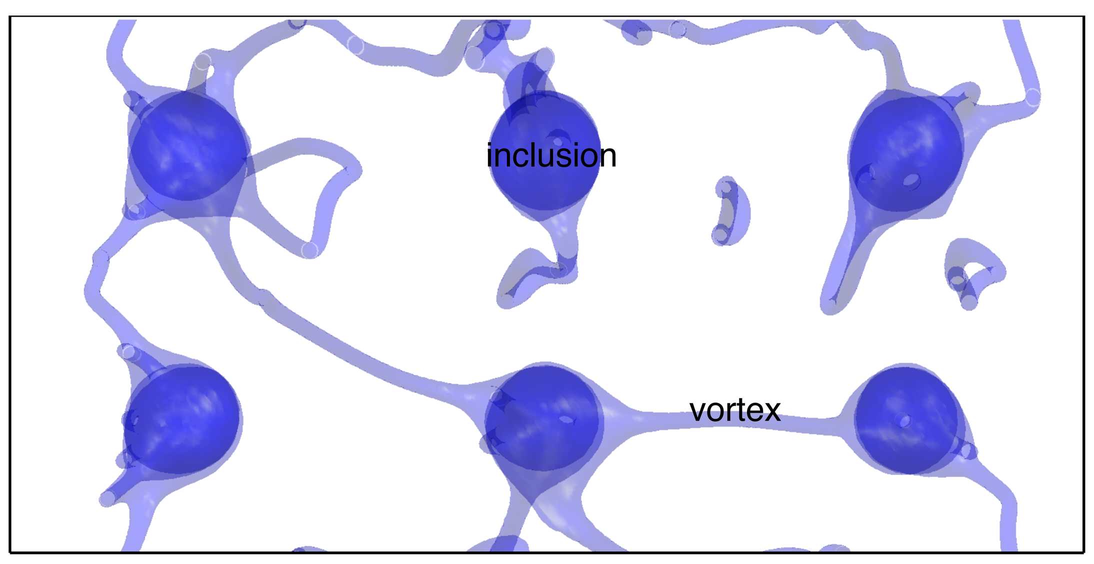
Type-II (high temperature) superconducting materials that have been engineered to sustain higher critical currents, transporting energy with little to no dissipation loss, have many applications.



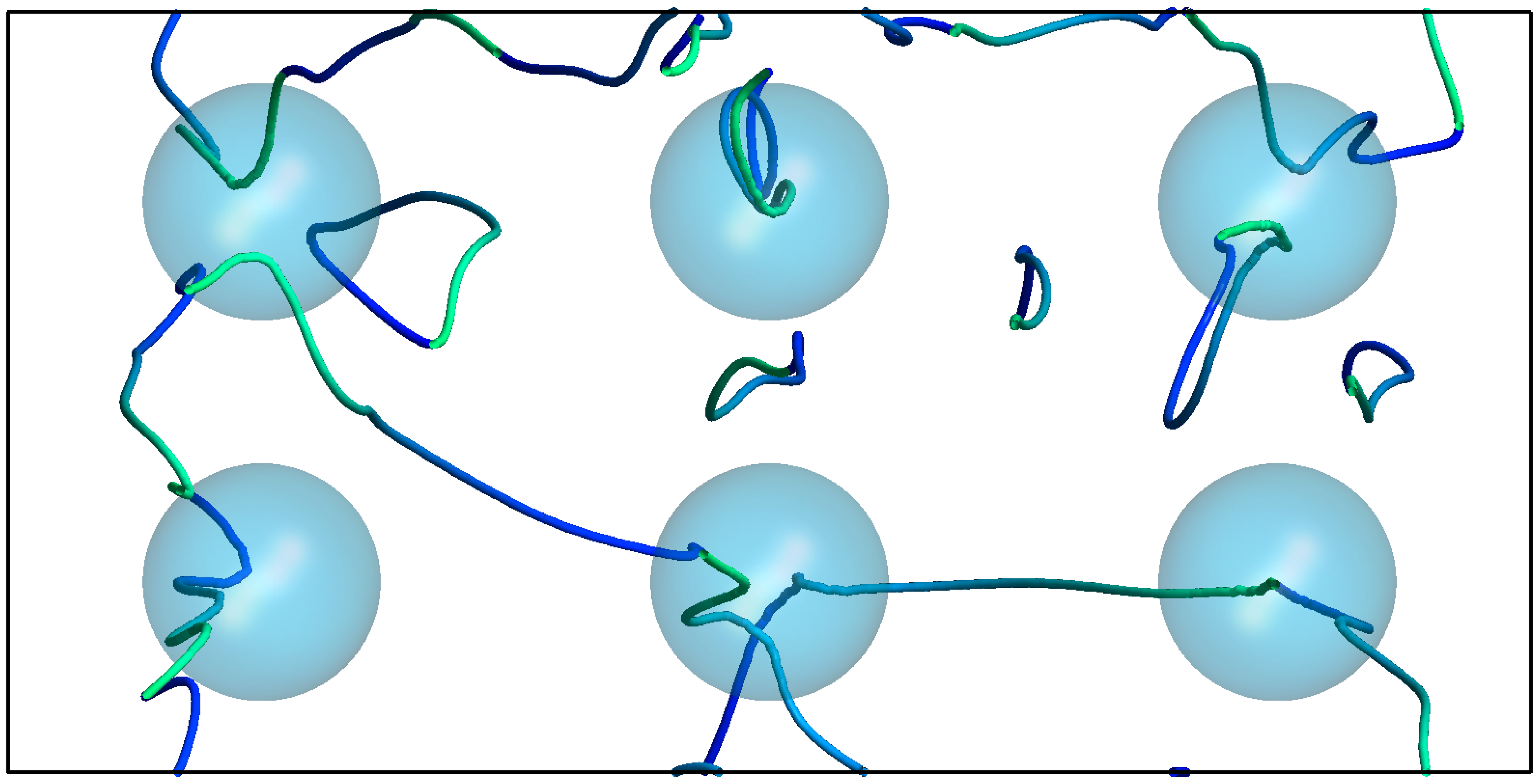
Large scale three-dimensional models of type-II superconductors using time-dependent Ginzburg Landau equations to model the evolution of the superconducting complex order parameter capture the dynamic processes inside of the material that determine the performance of the material.

Most important is the dynamics of magnetic **vortices**, flexible tubes carrying magnetic flux quanta that are born, merge, die, and pin and depin on material inclusions inside the superconductor.

Prior Analysis Method



Historically, these vortices have been detected from simulation data by examining contour plots and isosurfaces of the complex order parameter field. However, these methods are imprecise and blur the fine details of how vortices interact with inclusions and each other.



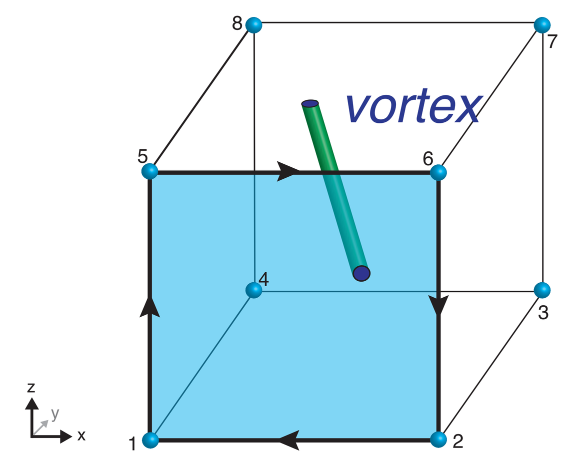
Physicists can now observe the emergent interplay of the vortices inside a model superconductor in higher resolution than has previously been possible.

New Analysis Method

Instead of mapping contours in the magnitude of the complex order parameter field, points on the vortex line core can be exactly determined by detecting quantized phase jumps in closed curve line integrals around the mesh element faces.

Under high magnetic fields, phase lines bunch together and the closed line integral generates errors. By applying a gauge transformation to the complex order parameter field, the phase lines are always the most dilute and the quantized phase jump can be accurately measured.

$$\underbrace{2\pi n}_{\text{phase jump}} = \oint (\underbrace{\nabla\theta}_{\text{phase angle}} - \underbrace{\boldsymbol{A}}_{\text{gauge transformation}}) \cdot dl + \underbrace{\int_S \boldsymbol{B} \cdot da}_{\text{magnetic flux}}$$



matrix element

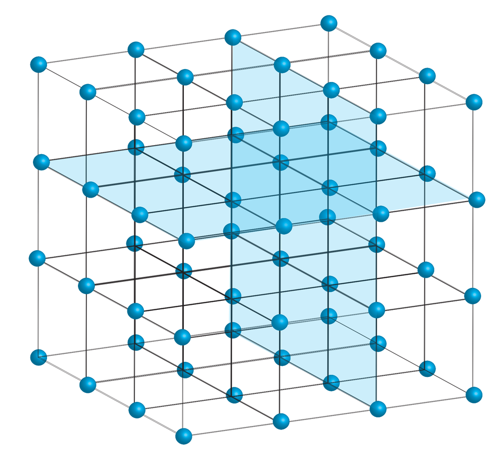
When a phase jump is detected in the line integral, an interpolation of the transformed complex order parameter on the face locates the point where the vortex line penetrates the face.

Future Project Goals

This method will be adapted to unstructured meshes and implemented in-situ in large scale parallelized simulations of superconductors. We are developing techniques to track vortices frame to frame. Then vortex dynamics can be detected and tracked and activity events recorded while the simulation is running.

Finite Element Model with a Structured Mesh

On a structured mesh, this feature extraction can be posed as a set of matrix operations performed on slices of the mesh. These can be implemented using optimized matrix algebra libraries. Using this method, the vortices can be trivially described at resolution even finer than the finite element mesh itself.



$$2\pi n = \sum \Delta\theta = D_{1-5} + D_{5-6} + D_{6-2} + D_{2-1} + dxdzB_y$$

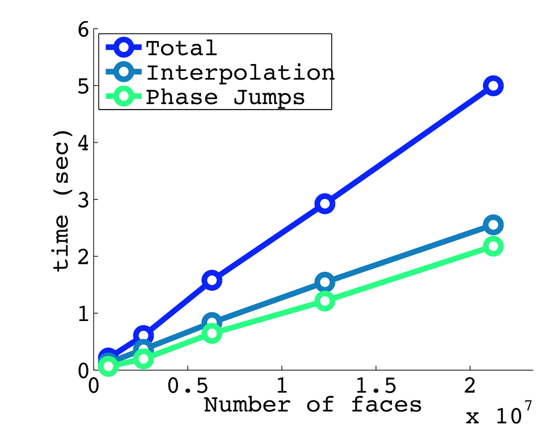
$A_j = \theta(:, j, :)$, an $n_z \times n_x$ slice of 3D array.

$$D_{1-5} = M_{2\pi}(D_z A_j + \{G^z_{xz}\}^{(\boldsymbol{yz})} + \{g^z_{xz}(j) J_{n_z,n_z}\}^{(\boldsymbol{xz})})$$
$$D_{6-2} = \text{Roll}(\text{-}D_{1-5}, (0,1))$$
$$D_{2-1} = M_{2\pi}(-(A_j D_x + \{g^x_{xz}(j) J_{n_z,n_x}\}^{(\boldsymbol{yz})} + dxKJ_{n_z,n_x} + \{QP^x_{xz}\}^{(\boldsymbol{yz})} + \{qp^x_{xz}\}^{(\boldsymbol{yz})}))$$
$$D_{5-6} = \text{Roll}(\text{-}D_{2-1}, (1,0))$$

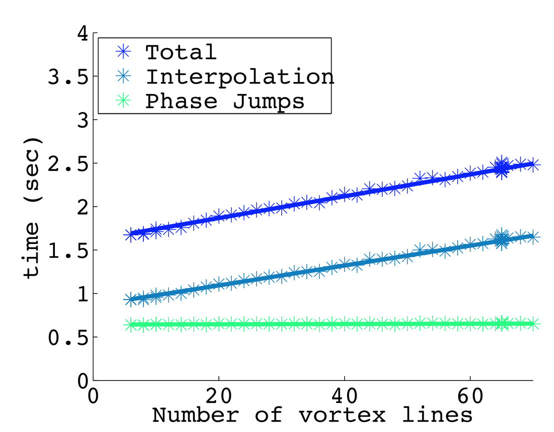
Performance

Vortices requires only ~1% of the memory of storing the entire field. Even more compact representations may be possible.

For serial python+numpy implementation:



Computation time increases linearly with the number of mesh faces in the simulation .



As the number of vortices increases, the calculation of the number of phase jumps stays constant while the interpolation computation time increases linearly

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